Markov chain modelling and ENSO influences on the rainfall seasons of Ethiopia

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Introduction

Ethiopia is one of the poorest counties in the world whose economy is highly dependent on rain-fed agriculture. Climate variability is assumed to be the main cause for the frequently occurring drought in Ethiopia. Nowadays, famine and the name of Ethiopia are highly associated. This is because, for countries like Ethiopia, meteorological drought (deficiency of rain with respect to meteorological means) and agricultural drought (deficiency of rain with respect to crop water requirement) are immediate causes of famine.

Accordingly, the National Meteorological Services Agency (NMSA) of Ethiopia should build its capacity and tailor its services in the way that decision-makers get benefit from its services. In fact, the National Meteorological Services Agency of Ethiopia is actively participating in the National Early Warning System aimed at mitigating the effects of natural disasters, such as drought. But the role of the Agency will be more beneficial if the information it provides is updated each time by results of various research activities.

As Ethiopia is located within the tropical region, it is influenced by weather systems of various scales, from mesoscale, such as thunderstorms, to large scale ENSO related phenomena. The major rain-bearing system for the main rainy season (June to September) is the Inter Tropical Convergence Zone (ITCZ). On the other hand, the eastward moving mid-latitude troughs will facilitate the interaction between the mid-latitude cold air and the tropical warm air so that unstable conditions will be created for the moisture that comes into Ethiopia from the Arabian Sea during the small rainy season (Feb. to May) (NMSA, 1996).

Most of the time, agricultural planning is difficult during the small rainy season (Belg) due to the erratic nature of the rains. Moreover, in association with ENSO phenomena, significant year-to-year variations in the performance of the rainy seasons has influenced the agricultural activities of the country. The forecasters of the NMSA of Ethiopia are aware of the problems associated with a reliance on forecasts using ENSO analogues. They have succeeded for the past several seasons during which such forecasts were issued. The government decision-makers are using their recommendations to alter agricultural practices on relatively short notices in order to maximize the value of the forecasted rains and minimize the impacts of forecasted droughts (Nicholls & Katz, 1991).

Recent advances in statistical methods have dramatically improved the range of techniques available for analysing data that are not from normal distribution. These new techniques, which are used in this study, parallel those used in the analysis of variance and regression for normally distributed data. This development is of considerable importance, since daily rainfalls are clearly not normally distributed. (Stern et al., 1982b).

Nowadays, fitting and testing a wide range of models for daily rainfall data is easy due to the wide availability of computer packages associated with these new techniques, particularly, the Instat package developed by the Statistical Services Centre of the University of Reading. Instat + for windows (Version 1.3.1 test) is used for the most of the analysis done in this work.

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The studies made by the Tropical Agricultural Meteorology Group of the University of Reading, at the end of the 1970s and the beginning of the 1980s, described the types of models that can be used to describe the chance of rain on a given date and rain per rain day. The results of their studies have become the basis for the studies done, in this area of research, since that time.

Due to data quality and missing data problems as well as the limitation of time, this work is done on the data obtained from the capital city of Ethiopia, Addis Ababa. Apart from its usefulness in agricultural planning, the result of this work can also be used in water resources management and construction activities. Based on the usefulness of the results obtained from this work, the analysis has been extended to other places such as, Kombolcha (a place in North-east Ethiopia) and Dire Dawa (a place in East Ethiopia).

Methods

The rainfall Model

Probability of rain

Given a record of n years and that a particular event occurred in m of these years, the probability of that event occurring in any given year is simply m/n. A curve is fitted to the probabilities using the computer package, Instat. Instat enables curves to be fitted to proportions which, by definition, are not normally distributed. As it was described in Stern et al. (1981), to ensure that the fitted curve is one of the probabilities (p) should be transformed as:

\[ f = \log\left(\frac{p}{1-p}\right) \]  

This allows f to vary from -\(\infty\) (p=0) to +\(\infty\) (p=1). A function \(F(t)\) (where t is the day of the year) is fitted to the values of f and the fitted probabilities are given by

\[ p(t) = \exp\left(\frac{F(t)}{1+\exp(F(t))}\right) \]  

For the function \(F(t)\) of Fourier series of n harmonics:

\[ F(t) = a_0 + a_1 \sin x + b_1 \cos x + a_2 \sin 2x + b_2 \cos 2x + \ldots + a_n \sin nx + b_n \cos nx \]  

where \[x = \frac{\pi t}{366}\].

This gives a function, which joins at the beginning and end of the year. This rather complex curve with 2n+1 coefficients was required here to describe the rainfall pattern.

For the rainfall that depends on the conditions of the previous dates, different curves were fitted to probabilities of rain for days falling into four classes of the 2\(^{nd}\) order Markov chain modelling:

- Yesterday dry previous day dry \(p_{rdd}\)
- Yesterday dry previous day wet \(p_{rrd}\)
- Yesterday wet previous day dry \(p_{rdr}\)
- Yesterday wet previous day wet \(p_{rrr}\)
Amounts of rain

Gamma distributions were fitted to amounts of rain on rainy days (zeros excluded). The gamma distribution is given by:

$$F(x) = \frac{k^k x^{k-1} e^{-\frac{kx}{\mu}}}{\Gamma(k)}$$

where $\Gamma$ is the gamma function and the two parameters, the mean rain per rain day ($\mu$) and the shape parameter $k$.

A Fourier curve was fitted to the mean amounts of rain per rain day. $k$ was assumed to be constant through the year and was estimated from all rainfall amounts.

Modeling of daily rainfall using Markov chains

General

If daily rainfall data obtained from meteorological observations is directly used for doing the data analysis, then it is called the direct method (Stern et al., 1982a). This kind of analysis has some limitations, such as:

- It needs many years data;
- It is difficult to change the results of the analysis on conditional basis (when the definition of the characteristics of interest is changed), in which case the analysis has to be done from the very beginning with the raw data.

For 30 years daily rainfall data used in this work, the direct method has given reasonably good results in describing the characteristics of interest. But, if the same analysis is planned for other rainfall stations of shorter data record, the method makes insufficient use of the data. Even for the meteorological station of our interest, if the rainfall data is categorized into different classes (such as years with El Niño, Normal and la Niña episodes), there will be a problem of insufficient data.

In order to incorporate short year records of rainfall in our analysis, an alternative method that involves modelling of daily rainfall data should be used. This method of data analysis is called the indirect method (Stern et al., 1982b). This approach, by making use of statistical models, summarizes a large volume of data as concisely as possible, by modelling daily rainfall using Markov chains.

Various people have used the Markov processes for modelling rainfall and have got very useful results. The indirect method enables fitting of Markov Chain models of different order into the probability of occurrence of rain and mean rain per rain days. The next section deals with fitting of these models into unconditional and conditional probability of occurrence of rains.

Probability of rain

Computation of probability of the rains is useful in the study of the distribution of rainfall in time. Studies done in different parts of the world show that the probability of getting rain at a particular time is dependent on conditions of the previous dates. These kinds of probabilities are called conditional. Before dealing with the conditional ones, it is good to show the overall chance of rain (unconditional probability) at Addis Ababa.
The overall probability of rain

The proportion of rainy days estimates the probability of rain, on any given date, at that place. Figure 1 shows the overall chances of rain and the zero order Markov chain fitted curve.

![Figure 1. Observed and fitted (0-order Markov chain) proportion of rainy days at Addis Ababa Bole based on 1964-2000 rainfall data. The fitted curve is a second order Fourier series with two sine, two cosine and constant terms.](image)

Figure 1 shows that the overall chance of rain in the main rainy season (June to September) is significantly higher compared with the small rainy season (Feb. to May).

Generally, the overall chance of rain and the fitted curve tells us that the chance of getting rain in the main rainy season is about twice as compared to the small rainy season. Such information is very important for agricultural planning.

Conditional probabilities

In some places of the world, it is found that the probability of getting rain on a particular date is dependent on whether some of the previous dates are wet or dry (Stern et al., 1982b). The 1st order Markov chain analysis deals with computation of chances of rain depending on whether a previous date was dry or wet.

Figures 2 and 3 show the conditional probability of rain at Addis Ababa Bole. Comparing figure 3 and 1, the probability of getting rain on a particular date in the small rainy season shows a significant increase, if it is followed by a wet day. On the other hand the probability of getting rain in the small rainy season will decrease significantly, if it follows a dry date. This condition suggests that the probability of getting rain in the small rainy season is significantly dependent on whether the previous date was dry or wet.

In order to check whether the probability of getting rain on a particular date is dependent on the conditions of the previous two days, a second order Markov chain analysis needs to be done. The appearance of figure 2 and 3 suggests that the second order Markov
chain analysis should be done, particularly for the small rainy season, as the probability of rain on a particular date in this season is strongly dependent on the condition of the previous date.

Figure 2. Probabilities of rain at Addis Ababa Bole depending on whether a previous date is dry or wet (based on 1964-2000 rainfall data). \( P_{rd} \) stands for probability of rain if the previous date is dry and \( p_{rr} \) stands for the probability of rain if the previous date is wet.

Figure 3. First order Markov chain fitted curves for probabilities of rain at Addis Ababa Bole (based on 1964-2000 rainfall data). \( f_{rd} \) stands for probability of rain if the previous date is dry and \( f_{rr} \) stands for the probability of rain if the previous date is wet. The fitted curve \( f_{rd} \) is 3rd order Fourier series with 3 sine, 3 cosine and a constant term. The fitted curve \( f_{rr} \) is a two and half Fourier series with 3 sine, two cosine and constant terms.
Figure 4. Second order Markov chain curves for probabilities of rain at Addis Ababa Bole (based on 1964-2000 rainfall data). \( f_{rdd} \) stands for probability of rain if the previous two dates are dry, \( f_{rdr} \) stands for the probability of rain if it is followed by one wet and one dry date consecutively, \( f_{rrd} \) stands for the probability of rain if it is followed by one dry and one wet date consecutively and, \( f_{rrr} \) stands for the probability of rain if the previous two days are wet. The fitted curve \( f_{rdd} \) is 3\textsuperscript{rd} order Fourier series with 3 sine, 3 cosine and a constant term. All the other fitted curves are 2\textsuperscript{nd} order Fourier series with 2 sine, two cosine and constant terms.

By comparing figures 4 and 3 we can conclude that four curves of second order Markov chains are sufficient to describe the probability of rain at Addis Ababa Bole. This is because the curves are relatively closer to each other in the second order (figure 4), even in the small rainy season, than in the first order (figure 3). However, the \( f_{rdd} \) curve of figure 4 shows very low probabilities in the small rainy season. The maximum probability in the small rainy season associated to this curve is about 12% where as the maximum probabilities associated with the other curves vary between about 40% to 55%.

This low probability in the small rainy season, associated to two previous dry days, tells us that dry spells are likely to persist in the small rainy season as compared to the main rainy season. The relatively higher probabilities associated with two previous wet days also tells us that the probability of getting rain in the small rainy season is strongly dependent on the two previous days’ conditions.

On the other hand, in the main rainy season the dependence of the probability of rain on the previous two dates’ conditions is less as compared with the small rainy season.

**Modeling the rainfall amount**

The amount of rain on rainy days is described by a gamma distribution. The gamma distribution has two parameters, \( \mu \), the mean rain per rain day, and \( k \), which dictates the shape of the distribution. The mean rain per rain day, \( \mu \), is computed and plotted in figure 5 for Addis Ababa Bole together with the fitted Markov chain curve.
Figure 5. Observed and fitted mean rain per rain day at Addis Ababa Bole based on 1964-2000 rainfall data. The equation of the fitted curve is of a third order Fourier series with 3 sine, 3 cosine and constant terms.

Generally, the fitted curve shows that the mean rain per rain day varies in time and has maximum values in the peak months of the two rainy seasons of Ethiopia. The maximum value, as it can be read from the fitted curve, is slightly higher than 9 mm per rain day.

The shape parameter k is assumed to be constant throughout the year and estimated as 0.92 for Addis Ababa Bole. As the value of k is very close to 1, the rain per rain day at Addis Ababa is approximately exponential.

Effects of ENSO on Rainfall and its Implication for Agriculture

General

There is a strong belief in the National Meteorological Services Agency (NMSA) community of Ethiopia that the ENSO phenomenon has a great impact on the climate of Ethiopia. Philander (1990) mentioned that El Niño events are associated with variabilities in rainfall in equatorial East Africa.

Points described in the preceding paragraphs indicate that analysis that investigates the relationship between the ENSO phenomena and the inter-annual variation of the characteristics described in the previous chapters is reasonable.

The current chapter, by making use of the Markov chain modelling techniques described in the preceding section, will investigate the effects of ENSO episodic events on the amount and distribution of rainfall as well as dry spell lengths within the rainy seasons.

In order to study the inter-annual variability, the rainfall data is classified into years of La Niña (cold episode with cold SSTA), normal (less SSTA anomaly) and El Niño (warm episode with positive SSTA). The classification used in this work is based on CPC (2000b). They have attempted to classify the intensity of each event by focussing on a key region of the tropical Pacific (along the equator from 150° to the date line). The process of classification was primarily subjective using reanalysed sea-surface temperature analyses produced at the
National Centre for Environmental Prediction/Climate Prediction Centre and at the United Kingdom Meteorological Office.

Their classification is in a season–by-season breakdown of conditions in the tropical Pacific. In this work the classification done on June–July–August (JJS) season is used, as the correlation between ENSO parameters and rainfall characteristics is relatively strong for our area of interest. Moreover, the changes of oceanic conditions are relatively slow that the SST condition of the selected season more or less represents the conditions of the preceding and succeeding seasons, unless the season is in transition from one episode to the other.

<table>
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<th>Events</th>
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Table 1. Table that shows years of the rainfall data classified in to ENSO episodes based on CPC (2000b). The La Niña and El Niño category include years with weak, moderate and strong events of the corresponding episode.

Based on the above condition the years of rainfall data that were used in this analysis fall into the three groups as shown in table 1. The next section deals with fitting of Markov chain models into the classified years and study the characteristics of rainfall in each episode.

**Cumulative probability of rain and dry spell length**

**Cumulative probabilities**

Markov Chain modelling technique is applied to each of the El Niño and La Niña data groups independently and 100 years of daily rainfall data is simulated for each group. The simulated data is then summarized into monthly and seasonal values to produce the cumulative probability curves of the kind presented in figures 6 and 7.

![Figure 6. Comparison of cumulative probability curves of the episodic events in the small rainy season at Addis Ababa Bole based on simulated rainfall data.](image)
The inter-annual variation of rainfall amount in the small rainy season is generally high during El Niño episodes.

The rainfall amount varies between about 50 to 450 mm in El Niño years. But the inter annual variation is relatively less in La Niña years, between about 40 to 250 mm only. There is a significant difference between the episodes in the cumulative seasonal rainfall amount. In La Niña years, the cumulative seasonal rainfall amount is significantly less than that of the other episode.

The inter-annual variation in rainfall amount in the main rainy season is relatively less as shown in figure 7. The inter-annual variation is similar (similar slope and almost proportional end points) in the two episodes. The seasonal cumulative rainfall amount of the main rainy season is less in all El Niño years and high in all La Niña years. It should be noted here that, like in La Niña episode in the small rainy season, the seasonal cumulative rainfall amount is significantly less in El Niño than the other episode in the main rainy season.

### Figure 7. Comparison of cumulative probability curves of the episodic events in the main rainy season at Addis Ababa Bole based on simulated rainfall data.

Agricultural planning needs further analysis on the simulated data in order to get information about events like dry spell lengths in a given rainy season. With the use of the values obtained from Markov Chain modelling, the Instat statistical package enabled us to compute the 10 days of dry spell length within a 30-day period from the beginning of each month. In order to investigate the effect of ENSO phenomena on the probability of getting a specified spell length in a given period of time, the computed probabilities of dry spell lengths were compared among the different episodes. The results of the comparison on a 10-day spell length are shown in figure 8.

Figure 8 shows the effect of ENSO on the probability of getting a dry spell length of 10 days within the rainy seasons. Indirectly, the figures show the bi-modal nature of rain in the study area. The two minima (excluding the La Niña condition in the small rainy season) correspond to the main rainy months of the two rainy seasons of the study areas.

The effect of ENSO is significant in the small rainy season, as a 10-day long dry spell length is not common in the main rainy season. Like in the analysis of the seasonal rainfall amount, the La Niña episode significantly increases the chance of getting longer dry spell
lengths (e.g. 10 day) in a 30-day period of time within the small rainy season. On the other hand, the risk of a 10-day dry spell in the small rainy season is less in El Niño years as compared to the other episode. The risk of having a 10-day dry spell in the small rainy season is the lowest in March in El Niño years. The risk again rises in the remaining rainy months of the small rainy season until it becomes maximum in May. In May and June, El Niño relatively increases the risk of having a 10-day dry spell as compared to the other episode.

![Figure 8](image_url)

**Figure 8. Unconditional probability of getting 10 days dry spell length within 30 days from the beginning of each month based on the fitted first order Markv chain probability values.**

**Discussion, Conclusions and Recommendations**

The results presented in preceding sections have linked the existing rainfall variability to a world-wide known meteorological event, ENSO. The link found between the ENSO phenomena and rainfall variabilities in Ethiopia is not a simple statistical chance. From personal experience of the author, works done for the purpose of seasonal weather forecasting purpose in the National Meteorological Services Agency (NMSA) of Ethiopia indicate the existence of strong link between these events. Moreover, work by Babu (1991), Tadesse (1994) and the recent work of Mekonnen (1998) also show the existing effect of ENSO on rainfall variability in Ethiopia.

The 1997/98 El Niño and the following three years’ La Niña events and the associated natural disasters in Ethiopia have created a great public awareness in Ethiopia regarding ENSO episodic events. There is a strong belief in the NMSA community of Ethiopia that El Niño reduces the rainfall activity of the main rainy season over drought prone areas and increases the chance of rains in the small season as well as the dry season of Ethiopia. On the other hand, the La Niña event is believed to affect the rainfall activity over the drought prone areas in the opposite way of El Niño. For example, recently, the reduced rainfall activity in Kiremt 1997 and the unusual heavy rainfall and associated flooding in the dry season of 1997/98 in Ethiopia was blamed on the El Niño event of that year. Moreover, the recent (1999/2000) wide-spread drought in Ethiopia and neighbouring countries was associated with the recent La Niña event. The effect of this La Niña event is in such a way that it has affected the rainfall activity of the small rainy seasons in three consecutive years.
During La Niña episodes special attention should be given to agricultural activities, as the failure of rain in this season also indirectly affects the agricultural activities planned for the main rainy season. On the other hand, during El Niño years, the risk of having less rainfall is for the main rainy season. It should be noted that according to these figures, the probability of getting rain increases in the small rainy season in El Niño years. Agricultural planners should seek ways of appropriate use of water resources in La Niña years for the small rainy season and in El Niño years for the main rainy season.

The inter-annual variability of rain in the small rainy season is associated not only to ENSO conditions but also to some other conditions. The rain-bearing systems of the small rainy season also include eastward moving mid-latitude troughs. As mentioned earlier, the rainfall activity of this season partially depends on the frequency and strength of these systems. Any event that affects the frequency and movements of these systems, such as blocking highs and omega systems over Europe, indirectly affects the rainfall activity in the small rainy season. Hence, agricultural planning should also take into account results of the studies done with regards to these events, in addition to the effects of ENSO events.

The increased rainfall activity in the main rainy season during La Niña years as well as increased rainfall activity in small rainy seasons during El Niño years and dry seasons of Ethiopia can have negative effects on agricultural activity. The associated heavy falls can destroy crops and also cause water-logging problems. On the other hand, the unseasonal rain in dry seasons during El Niño years may affect the harvesting activity.

The results shown in figure 8 also give additional information about the nature of the rains in different episodes. Consistent with the results discussed in the above, the La Niña episode increases the risk of elongated dry spells (10 days in this case) in the small rainy season. On the other hand the risk of persistent dry spells in the main rainy season is very low during the La Niña episode. Hence, apart from problems of decreased rainfall amount, the risk of dry spells makes the small rainy season very difficult in the planning of rain-fed agricultural activity.

During the El Niño episode there are slightly higher risks of dry spells of a 10-day period, in the main rainy season, particularly, in the period between mid-June to mid-July. This risk becomes relatively higher throughout the season. Based on the water requirements of a crop, appropriate action should be taken by decision-makers to overcome the relatively higher risks of elongated dry spells in El Niño years in the period between mid-June to mid-July.

The results of daily rainfall analysis discussed so far gave us very useful information for agricultural planning in Ethiopia, particularly for areas close to Addis Ababa. The overall results of the analysis are summarized by the conclusions presented in the next section.

Conclusions

The analysis done with use of Markov chain modelling techniques enabled us to use the rainfall data sufficiently. The Markov chain models are found to be fitting well and the results obtained from these techniques are also very useful for agricultural planning. More importantly, the Markov chain modelling techniques were successfully used in the study of effects of ENSO on the rainfall characteristics of the study area.

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Sometimes an anticyclone may form in the mid-latitudes well poleward of the normal position of the subtropical highs. The development is associated with a temporary halt to the normal eastward procession of the depressions in the mid-latitudes, with the result that it has become known as a blocking high.
Due to the existing ENSO signal on the rainfall activity of the study area, agricultural planners should be advised to incorporate the possible effects of ENSO episodic events in their decision-making.

**Recommendations for future studies**

The following future studies are recommended so that the results obtained from this work can be used in the operational works of the National Meteorological Services Agency of Ethiopia:

As it was mentioned in the previous section, daily rainfall analysis is found to be very useful for agricultural planning. Hence, this work should be extended to other drought prone areas and to all over the country at large. Regionalizing the characteristics and identifying risks in time and space with respect to agricultural activity are very useful for agricultural planning.

The rainfall characteristics of the two rainy seasons are found to be different. Markov chain modelling techniques applied to the individual seasons can be more useful in investigating further the characteristics of each season.

Both tropical and extra-tropical weather systems are involved in the rain-bearing mechanism of the small rainy season. Apart from the existing ENSO signal on the rainfall activity of the small rainy season, events that affect these extra-tropical weather systems indirectly affect the rainfall activity of this season. Studies that investigate the relationship between Blocking Highs and Omega systems over Europe and the rainfall variability in the small rainy season of Ethiopia will be very useful in this regard.

**References**


Mekonnen, A., 1998: Interannual Rainfall Variability of the Horn of Africa. *MSc. Dissertation, Department of Meteorology, University of Reading, UK.*


